Post yield phenomena in tensile tests on poly(vinyl chloride)

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Tensile tests have been made on poly(vinyl chloride) to find the conditions under which uniform extension, diffuse necking, localized necking or thermal fracture occur. These observations have been compared with the predictions of a finite element model of the tensile test specimen to ascertain the causes of the changes in post-yield behaviour with increased crosshead speed. The major effect of increased crosshead speed is to reduce the dissipation of heat generated by plastic deformation. The effect of thermal pre-treatment of the PVC is to change the extent of strain softening after yield, and thereby control the range of post-yield phenomena that are possible.

INTRODUCTION

The most characteristic phenomenon which occurs after the yield point in a tensile test with a thermoplastic is that of necking followed by cold drawing[1]. However not all plastics exhibit necking. The cellulose derivatives have long been known to extend uniformly and more recently it has been shown that the 1-nylons show a similar behaviour[2]. From the molecular viewpoint it is believed that uniform extension correlates with an extended chain configuration. From the physical aspect the condition for necking was first proposed by Considere[3] and applied to plastics by Vincent[1]. This condition states that:

$$\frac{d\sigma_{\text{true}}}{d\epsilon_{\text{true}}} < \sigma_{\text{true}}$$

or (identically)

$$\frac{d\sigma}{d\epsilon} < 0$$

where σ is the engineering stress, σ_{true} the true stress, and ϵ the strain.

With most plastics the necking mode is well established and occurs over a wide range of conditions. However, it now appears that the Considere condition is not fully adequate. For example, when $d\sigma/d\epsilon$ is negative but small, or where σ_y/σ_d (where σ_y is the yield stress and σ_d the drawing stress) does not greatly exceed unity, the mechanical advantage of the necking process may be insufficient to generate a stable neck[4].

Further it is quite common for specimens to fracture immediately after the neck is formed especially with high strain rates[1,5] or long test pieces[6] where pronounced local heating takes place in the first stage of neck formation. Even at low strain rates, however, it now appears that new types of crack initiation and propagation can take place after the neck is formed[7] which leads to early 'plastic fracture', although this contingency is not covered in the present treatment.

Thus polymers which extend in tension may show uniform extension, stable necking or fracture in an incipient neck. Some polymers show more than one of these phenomena, but it is rare that all three can be observed with a single polymer. It is shown here that unplasticized poly(vinyl chloride) (PVC) can exhibit all three phenomena, and the conditions under which each occurs are investigated.

If the mechanisms, which cause the transition from one type of post-yield behaviour to another, could be established, then it should be possible to specify more clearly the most appropriate tensile testing conditions for a particular use of a thermoplastic. It has been suggested, on the basis of correlation of data[8,9] that the viscoelastic relaxation time spectrum of PVC controls the variation of the average strain at rupture with test rate and temperature. A recent alternative approach[10] has been to put forward a semi-empirical constitutive equation for the deformation of a thermoplastic, and then to use numerical methods to compute the strain and temperature distributions that develop during a tensile test. This latter approach, as well as considering purely viscoelastic effects, incorporates effects that only become significant beyond the yield point, such as strain softening and heat generation due to plastic deformation[11,12]. It should therefore stand a better chance of predicting the phenomena that may occur.

One aim of this research has been to investigate the mechanical causes of the phenomenon of necking. Bowden[13] has suggested that the propensity of a glassy polymer to form shear bands, when compressed under plane strain conditions, is determined by the ratio of the minimum gradient of the stress—strain curve after yielding to the strain rate sensitivity of the yield stress. As indicated above, it is possible that a similar parameter might determine the propensity to neck formation in a tensile test.

THEORY

As the theroretical model has been explained in detail in ref 10, only the main features are summarized here. The tensile test specimen is simulated by a number of three-dimensional elements, that maintain displacement continuity along a single tensile axis. The strain, temperature and other variables are assumed to be constant within each element, and the one-dimensional connection in series means that the engineering stress is constant throughout the model. Each element has a constitutive equation that directly relates stress, strain and strain rate for the time interval being considered; and which takes account of the changes in element temperature, strain softening, and neck geometry that occurred in the previous time interval. The temperature distribution among the elements is calculated by a finite difference method, taking account of heating due to plastic work, thermal conduction from element to element, and thermal convection from the element surfaces in contact with air. Strain softening is taken to be a simple function of the accumulated plastic strain in the element. The inhomogeneous stress distribution across a real neck is averaged across the cross-section, and a correction factor for this effect is calculated by finite differences using the dimensions of the nearest neighbouring elements.

The complete constitutive equation for an element is.

$$F = [TF\{YS[1 - \exp(-Be)] + A\ln[(0.05 + e)/(0.1 + ST)]\}$$

$$+ TFF \cdot YS1 \cdot H] \cdot W \cdot GEOM$$

where F is the engineering stress in each element; $YS1$ is the initial yield stress at strain rate 1 sec^{-1} and 20°C = 74 (quenched) or 95 MN/m^2 (annealed PVC); A is the strain rate sensitivity $d\sigma_y/d\ln\dot{e}$ = 2.8 MN/m^2 (annealed) or 3.0 MN/m^2 (quenched); YS is the initial yield stress at the mean imposed strain rate; B is the constant in exponential approach to initial yield = 50; H is the strain hardening term = 0.12 × $[\exp(e - e_m - 0.05)^2 - 1]$; ST is the average strain increment per time interval, usually = 0.01 or 0.001; e is the strain in the element; e_m is the largest strain that the element has previously achieved.

The subsidiary functions, evaluated on the basis of the previous state of element are as follows.

(a) Thermal softening factors
TF for the plastic resistance part of the yield stress
= 1.0 − $\Delta T/76$ for $\Delta T < 17$
= $(51 - \Delta T)/42$ for $\Delta T > 17$
TFF for the strain hardening part of the yield stress
= 1.0 for $\Delta T < 20$
= $(120 - \Delta T)/100$ for $\Delta T > 20$
where ΔT = test temperature − 20°C
(b) Strain softening factor
$W = 1 - SL(e - 0.05)$
with a minimum value of $1 - DP$
where the slope SL = 1.0
and the maximum strain softening DP = 0.1 for quenched PVC and 0.4 for annealed PVC.
(c) Neck geometry factor $GEOM$. It was shown in ref 10 that the mean longitudinal stress, σ, in a neck deforming under plane strain conditions is approximately given by:

$$\sigma = Y[1 + a_i(a_{i+1} + a_{i-1} - 2a_i)/3x_i^2]$$

where Y is the plane strain tensile (or compressive) yield stress of the material, and a_i and x_i are the half-thickness and length, respectively, of the ith element. The multiplicative factor $GEOM$, defined as σ/Y, is evaluated from the above equation. Unfortunately unless the ratio of element length to thickness falls between the limits of 0.1 and 1 the necking process will not propagate satisfactorily. In the simulation described here the initial element length was 0.22 of the initial element thickness, the length being 0.67 mm, the thickness 3 mm, and the width 10 mm.

In the subsequent calculation of the temperatures of the elements it is assumed that all the plastic work is transformed into heat. The temperature rise is calculated assuming that the product density and specific heat remains constant at 1.42 MJ/m^3. The effects of heat diffusing from element to element, and convecting from the polymer−air interface are calculated[10] using a thermal diffusivity of 1.1 × 10^{-7}m^2/sec and a heat transfer coefficient of 9.1 Wm^{-2}K^{-1}.

It should be noted that an incipient neck is induced in the centre of the model by artificially increasing the engineering stress F on the central element by 1% until the strain in that element has reached 0.2. This is equivalent to saying that the cross-section of the simulated specimen has a 1% reduction in area at its centre initially.

As many as possible of the parameters in the constitutive equation were measured in experiments described in ref 10. The empirical parts of the constitutive equation are the strain rate term $A\ln((0.05 + e)/(0.1 + ST))$, which has no theoretical justification, and the neck geometry factor $GEOM$, which is theoretically valid only at the centre of a symmetrical neck. The strain softening factor W could only be estimated for quenched PVC, which deforms homogeneously in slow compressive tests. Hence the factor W was chosen empirically for the annealed, and the plasticized PVC, to give the best agreement with experiment. Finally it should be emphasized that the various parameters were evaluated in separate, simple tests, so it is not known whether the way in which the effects have been combined in the constitutive equation is correct.

EXPERIMENTAL

Materials and test methods

Two types of PVC were used in the experiments. The first was ICI Darvic 025 transparent sheet. This has a Fikentscher K value of 55 which implies that the viscosity-average molecular weight is about 30 000[14]. It contains less than 5% by wt of lubricants, pigments and soluble stabilizers. The sheet is manufactured by calendering sheets about 0.5 mm thick, which are then pressed together. It has a density of 1.39 g/cm^3 and a glass transition temperature of 67°C measured by a differential scanning calorimeter.

The plasticized PVC used was kindly provided by Dr P. R. Brooks of BX Plastics Ltd. It had a composition by weight: 100 parts PVC homopolymer of K value 55; 17 parts dialphanol* phthalate plasticizer; 1 part epoxy compound stabilizer; 1 part cadmium barium zinc liquid stabilizer. It had a glass transition temperature of 48°C measured at a heating rate of 16°C/min in a differential scanning calorimeter.

Tensile test specimens were cut from the 1 mm thick unplasticized PVC using a high speed router and a template. The specimen shape was similar to that specified in ASTM standard D638, type I, and had a gauge length of 60 mm and a width of 8 mm. Specimens were cut from the 1.7 mm
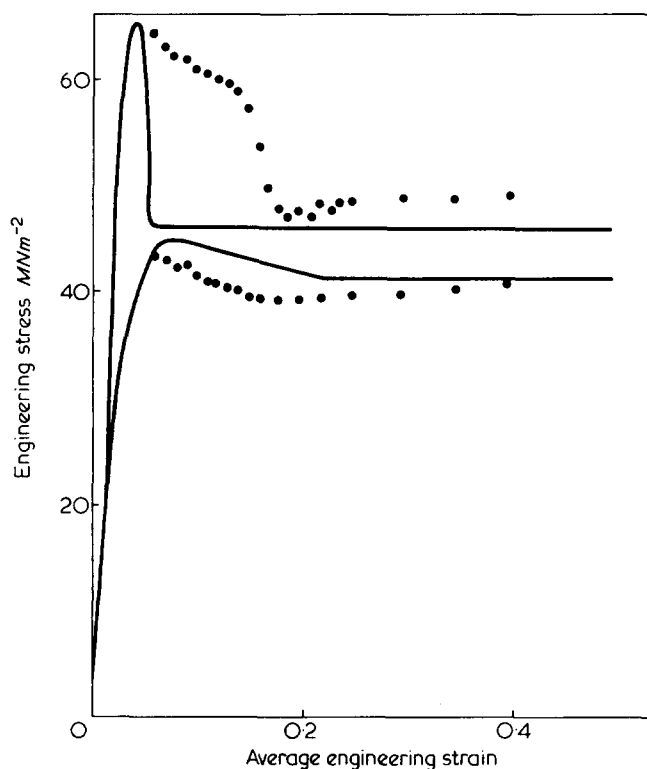
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* A mixed C$_8$ alcohol

Figure 1 Engineering stress *versus* average engineering strain for quenched and annealed PVC at 23°C at a crosshead speed of 0.5 mm/min. Solid curves — experimental data, uncorrected for test machine compliance, dotted curves computer simulation

thick plasticized PVC sheet using a metal die cutter; these ·had a gauge length of 30 mm and a width of 4 mm. The specimens were then given one of the three following thermal histories[15]:

(1) as received;

(2) quenched. The specimens were heated at 90°C in an air oven for 15 min, then quenched rapidly into iced water, and subsequently tested within 1 h of quenching;

(3) annealed. The specimens were heated to 65°C for 1 h in an air·oven, followed by 5 h at 50°C and 16 h at 40°C.

The tensile tests were performed with an Instron tensile testing machine, either in an environmentally controlled room at 23° ± 1°C and 50% relative humidity, or in a temperature-controlled chamber fitted to the machine.

RESULTS

If tensile tests on PVC are carried out at sufficiently low strain rates then the tests are effectively isothermal. *Figure 1* shows the difference in the recorded engineering stress—strain curves for annealed and quenched PVC under approximately isothermal conditions. For both materials there is an initial maximum in the stress (which we will subsequently call σ_y), followed by a subsequent minimum level (σ_D), but for the annealed PVC the difference $\sigma_y - \sigma_D$ is much greater. The annealed PVC was observed to form a local neck (*Figure 5a*) which then propagated, but the quenched PVC extended uniformly. The magnitude of $\sigma_y - \sigma_D$ for annealed PVC is similar to that found for the many other plastics which neck in tensile tests. The Figure also shows the simulated stress—strain curves using the parameters of the constitutive equation given earlier. The strain increment size *ST* employed in the model was set at 0.01 to economize on computation time, and this has the effect of delaying the formation of any neck to average strains much greater than

those observed. Nevertheless the types of necking behaviour are correctly predicted. The only significant differences in the model parameters between the annealed and quenched PVC are in the initial yield stresses *YS*1 and the extent of strain softening *DP* (= 0.4 and 0.1 respectively). The temperature distribution calculations show that the maximum temperature rises are 7.5° and 0.7°C respectively, and that this heat has diffused down the specimens to give a practically uniform temperature.

If the strain step size was reduced to 0.001 then it was predicted that the annealed PVC would neck, and the stress fall, at a mean strain of about 0.07, close to the observed value. However, a similar change in the strain step size lead the prediction that quenched PVC would also neck. It is evident that the strain step size affects the maximum feedback rate of the necking instability, and hence the type of extension predicted. This is a failing of the particular constitutive equation used. For PVC of a particular thermal pre-treatment, the necking behaviour depends on the imposed strain rate of the tensile test. *Figure 2* shows that the initial yield stress σ_y of annealed PVC increases linearly when plotted against log crosshead speed, the slope giving a measure
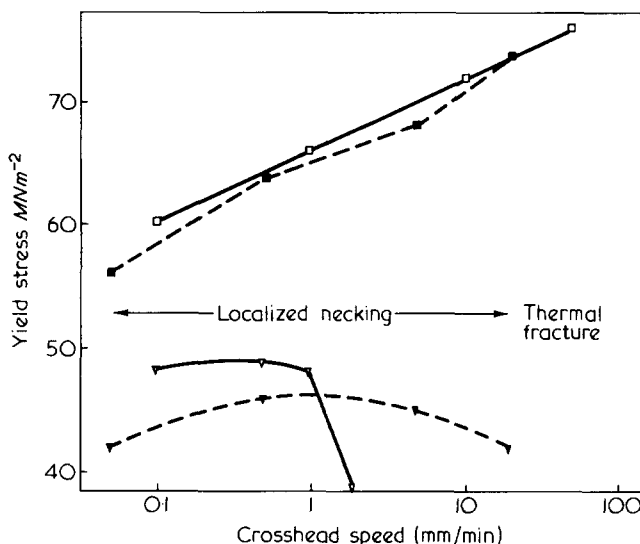

Figure 2 Variation of yield stresses with crosshead speed for annealed PVC at 23°C experimental σ_y (■), model σ_y (□), experimental σ_D (▼), model σ_D (▽). The type of necking observed is noted on the Figure
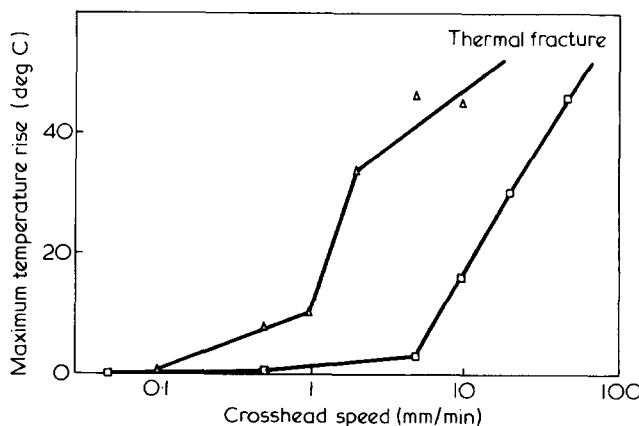

Figure 3 Precicted maximum temperature rises during tensile tests at 23°C on PVC specimens 50 mm long, 10 mm wide and 3 mm thick. △annealed, □ quenched PVC

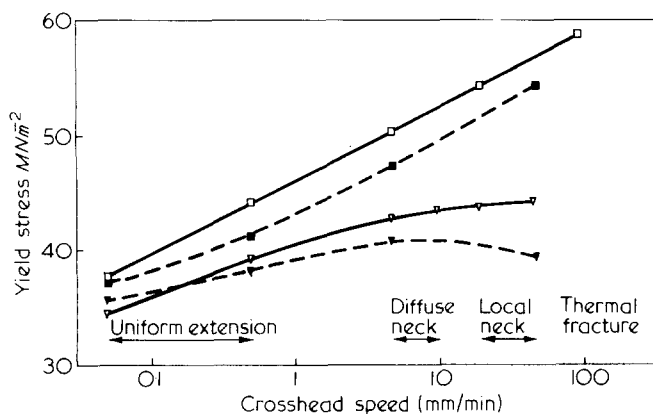*Figure 4* As *Figure 2* but for quenched PVC at 23°C

the graph. The incipient neck is then predicted to do one of three things.

(i) It may disappear, so that uniform extension occurs subsequently. For this to happen the value of *ERAT* must fall below 1, as occurs for a crosshead speed of 0.05 mm/min;

(ii) It may persist as a diffuse neck but the whole of the specimen continues to yield. For this to happen *ERAT* must stabilize at a value greater than 1;

(iii) It may develop into a localized neck, so that the ends of the specimen stop yielding. For this to happen the value of *ERAT* must increase to infinity.

It would appear from the results so far that the parameter σ_y/σ_D gives a reasonable indication of the type of necking behaviour observed, as values < 1.2 occur for uniformly extending specimens, and values > 1.4 occur when localized necks propagate. If this parameter is plotted
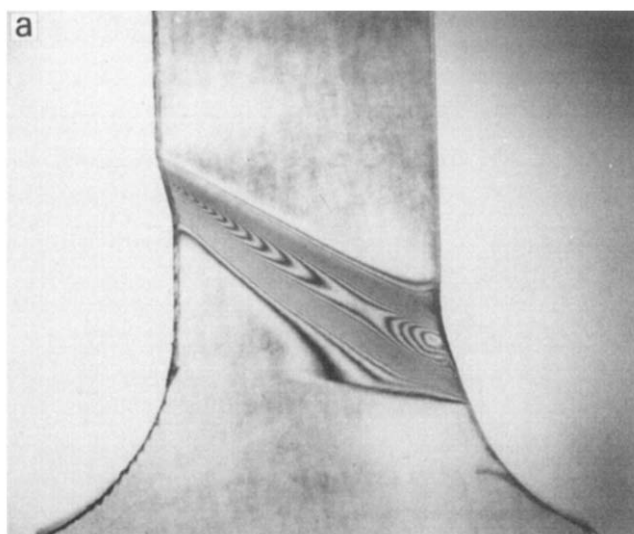
of the parameter *A*. However, the drawing stress σ_D shows a maximum at a crosshead speed of 0.5 mm/min. Localized necking (see *Figure 5a*) occurred at crosshead speeds up to and including 20 mm/min. At crosshead speeds of 50 mm/min and above, fracture occurred soon after neck formation. We infer from the computer simulation that this is the phenomenon of thermal fracture, since the temperature in the neck is predicted to exceed 70°C. The ratio σ_y/σ_D was observed to increase from 1.33 to 1.76 as the crosshead speed increased from 0.05 to 20 mm/min, and the computer simulation follows this fairly closely, even though the maximum strain softening *DP* remains constant at 0.4 and localized necking always occurs. It is clear that the ratio σ_y/σ_D depends on a number of factors, other than the strain softening, in particular the temperature in the neck, and the strain in the neck. This latter was predicted to increase from 1.04 at 0.1 mm/min crosshead speed to 1.43 at 10 mm/min crosshead speed, and the predicted maximum temperatures are shown in *Figure 3*. The experimental neck strains are known to average 1.2 [6].

Figure 4 shows similar results for PVC quenched prior to tensile testing. In this case a complete spectrum of necking behaviour is exhibited as the crosshead speed increases (see *Figure 5b*), the type of neck being noted in the Figure. The computer simulation, using a single set of parameters, predicts quite closely the variation in σ_y and σ_D, and also correctly predicts the type of necking. Note that, as the strain increment is held constant at 0.01, the varying time interval (or strain rate) only affects the constitutive equation directly via the initial yield stress *YS*. Hence, the predicted change in necking behaviour is largely governed by the temperature distribution which develops in the specimen.

In order to follow the development of the neck a strain rate parameter *ERAT* was calculated. This is defined by:

$$ERAT = \frac{\text{Strain increment in the central element}}{\text{Strain increment in the first element}}$$

$$= \frac{\text{Strain rate in the centre of the developing neck}}{\text{Strain rate in the unnecked region}}$$

If the ends of the specimen cease to extend then *ERAT* increases to infinity, and subsequent values of *ERAT* have no meaning. In *Figure 6 ERAT* is plotted against the mean strain in the specimen. The initial increase in *ERAT* is a result of the 1% cross-sectional decrease in the centre element. This artifice is removed when the strain in the central element reaches 0.2, at mean strains indicated by arrows on

Figure 5a Localized neck in annealed PVC immediately after formation, photographed in circularly polarized light to reveal the plastically deformed region. The isochromatic fringes give an approximation indication of the plastic strain
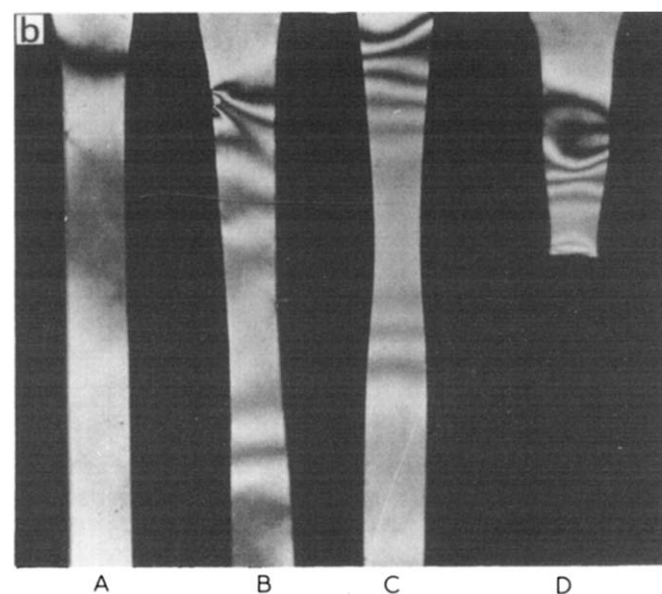

Figure 5b As *Figure 5a* for quenched PVC, during extension at 23°C at the crosshead speeds of A, 0.5; B, 5; C, 50 and D, 100 mm/min
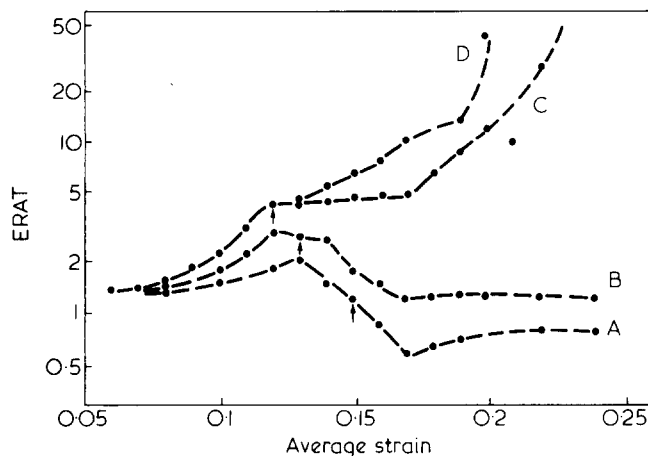
Figure 6 Development of necking instability *versus* mean strain for model of quenched PVC at 23°C, at the crosshead speeds of A, 0.5; B, 5; C, 50; and D, 100 mm/min. *ERAT* is the ratio of the maximum strain rate in the incipient neck, to that at the end of the specimen. The arrows indicate the mean strain at which the 1% extra load on the centre element is removed
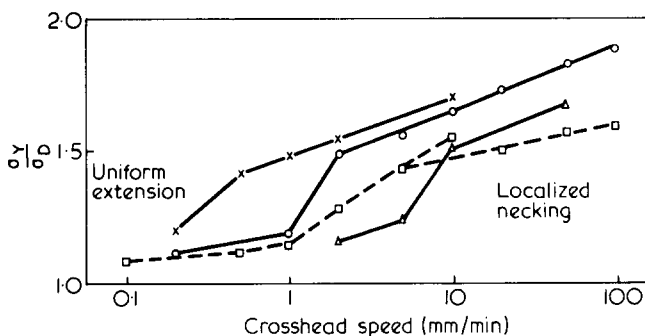

Figure 7 Ratio of initial yield to drawing stress *versus* crosshead speed for as-received PVC at 52°C (x), 60°C (O), 65°C (△) and for 17% plasticized PVC at 23°C (□)

against the crosshead speed on a logarithmic scale, then the transition in necking behaviour is clearly indicated. *Figure 7* shows such a plot for thermally untreated PVC tested at 52°, 60° and 65°C. The transition in necking behaviour moves from 0.4 to 8 mm/min crosshead speed as the temperature is increased in this range. It was also possible to observe[16] the transition in necking behaviour for the 17% plasticized PVC tested at 23°C, as is shown in *Figure 7*. The finite element model was run to try to simulate the data for thermally untreated PVC at 52°C, using the same parameters that were used at 23°C, except that the strain softening parameter *DP* was reduced to 0.2. The preaictons are summarized in *Table 1*.

The predicted variation of σ_y/σ_D with crosshead speed matches the observed values at 60°C reasonably well. The predictions do however provide a warning that the value of σ_y/σ_D need not necessarily mean a certain type of necking behaviour, since at a crosshead speed of 1 mm/min localized necking occurs, albeit with a low necking strain, for σ_y/σ_D = 1.19.

In order to show how the neck geometry and the temperature distribution can interact in tensile tests on PVC, *Figure 8* shows the computer simulations for quenched PVC at crosshead speeds of 10 and 50 mm/min, after a mean strain of 0.4. Only half the neck is shown because the results are symmetrical about the midpoint of the neck. It can be seen how great is the increase in temperature rise when a sharp neck forms, and how the necked material cools with the time elapsed after necking. Previous direct measure-

ments[17] of the temperature distribution on the surface of a PVC tensile specimen have shown how the unnecked material is hardly heated at a crosshead speed of 10 mm/min, whereas the temperature gradient in the necked material is quite gentle. Further work is in progress[18] to measure the temperature profiles in more detail.

One curious feature about the necking of PVC is that at a crosshead speed of 10 mm/min it was noticed that the neck propagated under certain conditions in one direction only[17] even though it started at the centre of the specimen. This implies that the single growing neck, propagating at twice the velocity that two propagating necks would have, is somehow more stable. In an attempt to produce this effect with the finite element model, the initial shape irregularity was modified so that the stress on the centre element was raised by 1%, and on its right hand neighbour by 0.5%, until the strain in each reached 0.2. This small asymmetry failed to make the neck propagation unidirectional except in the case of annealed PVC, extended at 23°C at crosshead speeds of 2 and 5 mm/min (there was slight asymmetry at 1 and

Table 1 Predicted behaviour of a thermally untreated PVC tested at 52°C. Strain softening parameter *DP* = 0.2

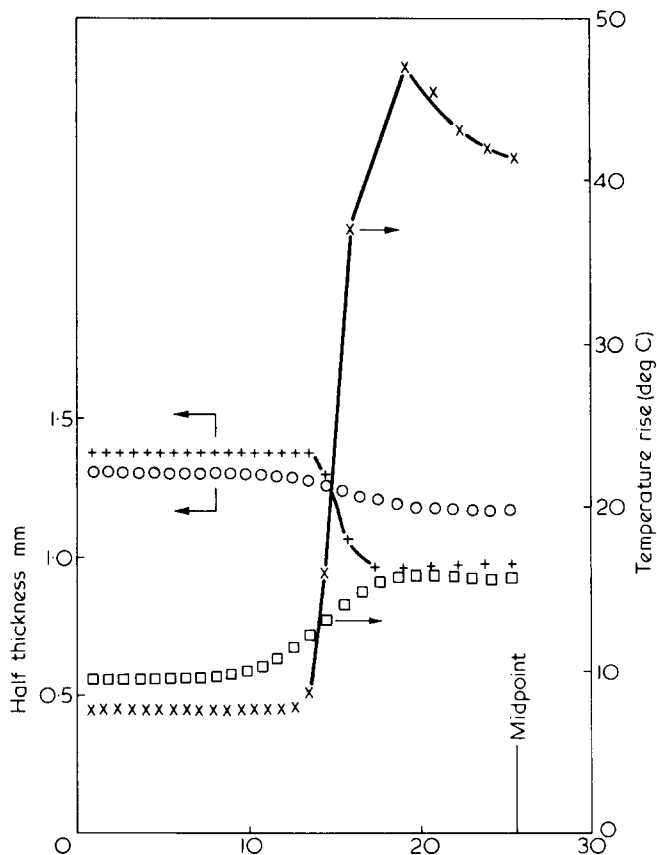| Crosshead speed (mm/min)	σ_y (MN/m²)	σ_D (MN/m²)	$\dfrac{\sigma_y}{\sigma_D}$	Maximum temperature in neck (°C)	Strain in neck
0.1	18.6	15.5	1.20	52.1	Uniform extension
1.0	21.5	18.1	1.19	54.0	0.6
10.0	24.4	15.1	1.61	62.0	0.9
50.0	26.6	17.9	1.49	67.5	1.15

Figure 8 Specimen half thickness and temperature rise *versus* length computed for quenched PVC at 23°C after a mean strain of 0.4. Crosshead speed 10 mm/min thickness (O), temperature (□); 50 mm/min thickness (+); temperature (x)
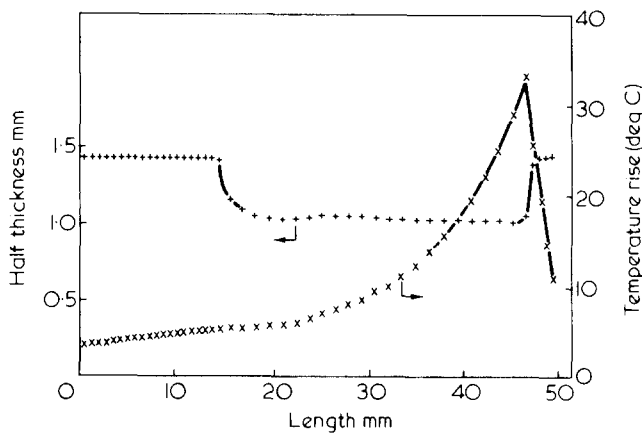
Figure 9 As *Figure 8* for a neck propagating to the right only in annealed PVC at 23°C and 2 mm/min crosshead speed, half thickness (+) temperature rise (x)

10 mm/min). *Figure 9* shows the neck shape and temperature profile for the 2 mm/min crosshead speed. The occurence of one-way neck propagation explains the sudden increase in the temperature rise in *Figure 3* at a crosshead speed of 2 mm/min. It would appear that in certain special circumstances a change over to one-way neck propagation produces a sufficient increase in the neck temperature that the drawing stress falls. The one-way propagation would not be expected to occur at low crosshead speeds when the temperature rises are small, or at high crosshead speeds when the temperature rise in the neck is effectively adiabatic.

DISCUSSION

In order to show that the viscoelastic relaxation time spectrum of PVC is responsible for the changes in the elongation to break as the tensile testing speed is varied, a means of making quantitative predictions must be established. Qualitative comparison of the relaxation time spectra, and the elongation *versus* time to rupture curves, at two temperatures[8] does not establish a relationship. One way of establishing the relationship could be as follows: stress relaxation data consists of plots of the relaxation modulus *versus* log (time) for various temperatures of measurement. It should be possible to produce a stress relaxation master curve (approximately at least) by shifting the data by a factor $a(T)$ along the log (time) axis. This is equivalent to saying that the longer time parts of the relaxation time spectrum shift by a common factor $a(T)$ when the temperature T changes. If the viscoelastic relaxation processes determine the elongation to break variation, then the same shift factors $a(T)$ should apply to the elongation to break *versus* log (time to rupture) graph. Examination of the published data[8] shows that the relaxation time spectrum shifts by 2 decades to shorter times when the temperature is increased from 23° to 52°C; however, the elongation to break graph only shifts by 0.6 of a decade. Therefore it seems most unlikely that the relaxation time spectrum directly determines the elongation to break. At 23°C the midpoint in the transition region of the elongation to break curve ($e_R = 0.5$) occurs at a time to rupture of 100 sec. For the 100 mm long specimens tested this means a crosshead speed of 30 mm/min. By reference to *Figure 3* we can see that this is the crosshead speed at which thermal effects are beginning to influence the rupture process. Thus it seems far more likely that the transition in elongation to break as tensile test speeds rise is caused by

thermal effects than by the secondary relaxation processes that occur in low strain deformation. The only relaxation process that would seem to be important is the main glass transition process. We have seen that a 17% plasticized PVC tested at 23°C, and an unplasticized PVC tested at 60°C show approximately the same changes in necking behaviour with tensile testing speed. This is presumably because the yield stresses are of the order of 20 MN/m², and the temperature dependence of yield stress is the same in the two cases.

If thermal effects play a large part in the reduction in the elongation to break of PVC tested at high speeds, then it is possible that the phenomenon of 'thermal fracture' discussed by Cross and Haward[6] is responsible. They introduced this concept for describing what happened when very long and thin specimens (typically 500 mm by 1 mm) were extended at low crosshead speeds (2 mm/min). Under these conditions, when a neck formed, the elastic unloading of the rest of the specimen was enough to cause rapid adiabatic heating of the neck, and fracture followed either by cracking or melting in the neck. The condition for this to be possible was that the specimen length L obeys:

$$L > Ex/(\sigma_Y - \sigma_D)$$

where E is the Young's modulus, and x, the extension due to the formation of a localized neck, is approximately equal to the product of the specimen thickness and the strain in the neck. This formula predicts that such 'thermal fracture' should not occur in the shorter, thicker specimens used in the work described here. Examination of the predictions of the finite element model lead to the following conclusions:

(A) Thermal fracture in the sense used by Cross and Haward does not occur for the 60 mm long specimens used here at a crosshead speed of 2 mm/min. It is predicted that stable necking occurs with a temperature rise of about 30°C (*Figure 3*).

(B) If the model is used for a 500 mm long by 1 mm thick specimen extended at 2 mm/min it predicts that 'thermal fracture' will occur when the neck temperature exceeds T_g.

(C) 'Thermal fracture' in another sense also occurs in the 60 mm long specimens when extended at rates > 50 mm/min at 23°C. This overheating is due to an attempt to propagate a neck too quickly, rather than forming a neck too quickly, since insufficient elastic unloading occurs for the latter to occur. Thermal fracture in this sense is akin to 'thermal runaway' in the fatigue testing of unnotched polymers[19].

A starting point for the discussion of the transition between uniform extension and necking is the condition[20] that has been put forward for the necking instability to occur

$$\sigma > \frac{d\sigma}{d\epsilon} = \frac{\partial\sigma}{\partial\epsilon} + \frac{\partial\sigma}{\partial\dot{\epsilon}}\frac{d\dot{\epsilon}}{d\epsilon} + \frac{\partial\sigma}{\partial T}\frac{dT}{d\epsilon}$$

where σ is the true stress, ϵ is the true strain, $\dot{\epsilon}$ is the strain rate and T the absolute temperature. In terms of engineering stress this inequality becomes that statement that necking will occur if the engineering stress should fall below its maximum value. This is inapplicable to PVC since uniform extension can occur in spite of a small fall in stress ($\sigma_Y/\sigma_D \sim 1.15$). However, the expansion of the differential in the above equation brings home the point that no single factor is responsible for the transition to necking behaviour. If, as with quenched PVC at 23°C, the material is balanced on the edge of the transition to necking, then it is possible that any variable that is altered will cause the transition. In particular an increase in the crosshead speed will increase the factor

$dT/d\epsilon$ by reducing the time available for convective cooling or thermal diffusion. Bowden proposed[13] that if $d\sigma/d\epsilon$ is large and negative compared with $d\sigma/d\ln\dot{\epsilon}$ then shear bands should form in plane strain compression tests. This is equivalent to saying that the combination SL/A, of the parameters of the constitutive equation, should be large to cause strain inhomogeneities. This cannot be the complete condition because it neglects any thermal effects. Furthermore the computer model runs suggest that it is the total amount of strain softening, rather than the rate of strain softening SL, which may be more important in determing whether or not necking occurs. This situation is further complicated by the existence of a half-way situation between uniform extension, and a neck propagating into material that is not plastically deforming. It has been observed that the strain can become inhomogeneous while the whole specimen continues to yield, and the computer model confirms this possibility. We have called the phenomenon diffuse necking. This has also been observed for ultra-high molecular weight polyethylene[4].

Finally the value and limitations of the finite element model for predicting the necking of PVC will be discussed. The advantage is that a single set of parameters, nearly all of which can be determined experimentally, can be used to predict the behaviour over a range of specimen sizes, crosshead speeds and test temperatures, the predictions bearing a close resemblance to the observed behaviour. It must however be admitted that the form of the constitutive equation was chosen so that the predictions were as close as possible to the experimental results. The inadequacies of the model can be discussed under three headings. Firstly there is the oversimplification inherent in the finite element model. The ratio of element thickness to length must be chosen to give a reasonable magnitude for the neck geometry effect, but this is set against the great reduction in computing time in using a one dimensional finite element model (~60 sec per simulation using a ICL 1906A), compared with using a two dimensional finite element model. Certain features of the three dimensional nature of neck formation and propagation in specimens cut from sheet have had to be neglected. For instance *Figure 5a* shows that a localized neck forms at an angle of $55°$ to the tensile axis; this makes the plastic strain increments in the neck compatible with the previous strains[20]. The edge of a propagating neck may be straight or may be curved, and it is usually aligned at $90°$ to the tensile axis. The propagation of a localized neck often appears to occur in two stages; first there is the localized contraction in sheet thickness, then there is a more diffuse contraction in sheet width which allows all of the necked material to reach the same plastic strain[21]. Secondly there are the uncertainties in the analysis of thermal effects (particularly convection), and the approximation in neglecting temperature variation within the elements. It would appear from the limited number of measurements of temperature distributions during necking made so far, that the predicted temperatures are somewhat too high. This may be because measurements were made on 1 mm thick sheet, whereas the calculations were for a 3 mm sheet thickness having a greater thermal capacity/surface area; or it may be because a fraction of the plastic work is not converted into heat but is stored in the yielded material; or it may be because the conditions of thermal convection cooling during a tensile test are different from those, in still air in a static sample, used to determine the heat transfer coefficient. Further work is in progress to clarify this point.

Thirdly there are the unjustified parts of the constitutive equation, in particular the basic strain rate dependence term $A\ln((0.05 + e)/(0.1 + ST))$. This neglects the known viscoelastic behaviour of polymers, and leads to a model in which the choice of the strain increment size ST affects the rate of neck formation. Furthermore, although the model illustrates how a positive feedback of a shape disturbance in a tensile specimen can lead to the formation of a localized neck, these events are predicted to occur at too high a mean strain.

It can be concluded that any analysis of the necking of polymers in tensile tests must include thermal effects in the constitutive equation. The constitutive equation suggested here is a reasonable first attempt, but needs improvements to be able to deal more effectively with viscoelastic effects. It is not possible to isolate a single material parameter that controls the rate of neck formation. However it appears that the total extent of strain softening is important in determining the type of necking behaviour in slow tensile tests. This parameter appears to be much smaller for quenched PVC than it is for annealed PVC, so that the former material can extend uniformly at slow tensile tests whereas the latter cannot.

REFERENCES

1   Vincent, P. I. *Polymer* 1960, 1, 7
2   Owadh, A. A., Parsons, I. W., Hay, J. N. and Haward, R. N. *Polymer* 1978, 19, 386
3   Considere, M. *Ann. Ponts Chaussees, Mem. et Documents, 6 Ser.* 1885, 9, 574
4   Trainor, A., Haward, R. N. and Hay, J. N. *J. Polym. Sci. (Polym. Phys. Edn)* 1977, 15, 1077
5   Vincent, P. I. in 'Physical basis of yield and fracture' Institute of Physics Conference Series, London, 1966
6   Cross, A. and Haward, R. N. *J. Polym. Sci. (Polym. Phys. Edn)* 1973, 11, 2423
7   Cornes, P. L. and Haward, R. N. *Polymer* 1974, 15, 149
8   Oberst, H. and Retting, W. *J. Macromol. Sci. (B)* 1971, 5, 559
9   Retting, W. *Eur. Polym. J.* 1970, 6, 853
10 Mills, N. J. *Br. Polym. J.* 1978, 10, 1
11 Binder, G. and Muller, R. H. *Kolloid Z.* 1961, 177, 129
12 Marshall, I. and Thompson, A. B. *Proc. Roy. Soc. (A)* 1954, 221, 541
13 Bowden, P. B. *Phil. Mag.* 1970, 22, 455
14 Koleske, J. V. and Wartman, L. H. 'Polyvinylchloride' Macdonald Technical and Scientific, London, 1969, p 27
15 Mills, N. J. *Eng. Fracture Mech.* 1974, 6, 537
16 Cross, A. *PhD Thesis* University of Birmingham (1977)
17 Cross, A., Hall, M. and Haward, R. N. *Nature* 1975, 253, 340
18 Maher, J. *et al.* submitted for publication
19 Andrews, E. H. in 'The Physics of Glassy Polymers' (Ed. R. N. Haward) Applied Science Publishers, London, 1973, p 411
20 Backofen, W. A. 'Deformation Processing', Addison Wesley Publ. Co. 1972
21 Walker, N. Personal communication